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Singularity Detection in Machinery Health Monitoring Using Lipschitz Exponent Function

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Abstract

Machinery health monitoring is a key step in the implementation of Condition-based Maintenance in industry. In this procedure, a quantitative description of machine health condition is necessary for maintenance decision-making. In this paper, we applied singularity analysis with wavelet for data processing and a new concept, Lipschitz exponent function, was proposed based on wavelet transform. A kurtosis based health index was defined, which can be used for maintenance decision-making. The proposed method was validated with two sets of gearbox vibration data in comparison with three other indexes. The results show that kurtosis based health index demonstrates excellent performance.

Keywords: Condition-based maintenance; Singularity analysis; Wavelet; Lipschitz exponent; Kurtosis; Health index

1. Introduction

In modern manufacturing industry, the role of machinery maintenance activities become very important, especially for those large-scale and important equipments and infrastructures, whose failures may result in significant economic losses or even catastrophic damages to human and society. Thus the research on optimal maintenance decision-making attracts interests of academia and industry. From the history of maintenance engineering, traditional maintenance strategies, such as run-to-breakdown and time-based preventive maintenance, are widely applied since they are easy to implement. However, these strategies are non-economical and they may lead to over-maintenance or under-maintenance in the process of implementation. Condition-based maintenance (CBM), which supervises maintenance activities on the basis of machinery health condition or status, is accepted as new research direction in this field. The successful implementation of CBM depends on accurate identification of health condition of machinery, which is realized through condition monitoring system.

Condition monitoring means identification of health condition with information collected from machinery. Generally, the information used can be vibration, acoustic emission, temperature, etc. Since vibration data are easy to collect and they contain abundant information related to machinery, a lot of researches have been conducted in the area of condition monitoring and summaries of them can be found in a recent review (Jardine et al., 2005). Figure 1 is a simple demonstration of flow chart of CBM program and from this graph we can see that a CBM program consists of four key steps:

1. Data acquisition unit, to collect data relevant to system health condition;

2. Data processing unit, to analyze collected data or

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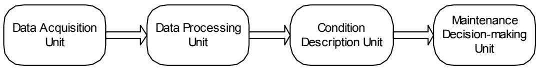

Fig. 1. Flow chart of CBM program.

signal so as to extract the most important information contained in raw data;

3. Condition description unit, to characterize health condition of system based on the results in Step 2;

4. Maintenance decision-making unit, to recommend maintenance activities and make decision on what to do under current condition.

The analysis of flow chart tells us data processing and condition description are the two main tasks in condition monitoring after data acquisition. Now, there are various kinds of techniques for data processing, including frequency based (Kuoppala et al., 1991), energy based (Al-Balushi and Samanta, 2002), and time-frequency based methods (Dalpiaz and Rivola, 1997; Yang et al., 2006). Here, we intend to introduce a new data processing technique, which is based on singularity analysis of signal with wavelet transform (Mallat and Hwang, 1992). Since we know that, for a rotating machine, the existence of faults produces singularities in collected vibration signals, singularity analysis with wavelet is a topic that attracts attention of many researchers recently. It has been widely applied in many areas including gear and bearing fault diagnosis (Loutridis and Trochidis, 2004; Sun and Tang, 2002), structural health monitoring (Robertson et al., 2003), damage detection (Hong et al., 2002) and even financial time-series processing (Struzik, 2001). However, most of these researches are to establish a fault diagnosis system utilizing pattern recognition techniques, such as artificial neural network, and they are not appropriate for CBM decision-making. The output of these systems is the conclusion of current health condition of machinery (e.g., normal, warning and failure), which is a discrete parameter with finite values and cannot help us to understand the exact status of machinery. For CBM decision-making, it is better to define a health index that can give a quantitative description of machinery health status. That is the task of condition description unit. Further, with such an index, we can conduct research on machine remaining life prediction using trend analysis and regression analysis techniques, which is important for machinery maintenance.

The main purpose of this paper is to propose a novel method for the construction of condition monitoring system. With this system, a quantitative description of machinery health condition is exported and it can be utilized for maintenance decision-making. Firstly we propose a data processing technique based on singularity analysis with wavelet and the output of this step is Lipschitz exponent function. After that, a health index is extracted from Lipschtiz exponent function. This research is the expansion of our previous research (Miao and Makis, 2005, 2007) and a comparative study will be conducted to demonstrate the new method.

The structure of the paper is as follows. Section 2 gives a brief introduction of singularity analysis and Lipschitz exponent function is proposed. Section 3 defines a new health index based on the result of Section 2. In Section 4, a comparison is conducted with two sets of gearbox vibration data. Conclusions and future work are summarized in Section 5.

2. Singularity analysis with wavelet: A new data processing technique

The wavelet transform of a function $f(t)$ is the convolution of the function with shifted and scaled versions of a wavelet function $\psi(t)$ and can be defined as (Mallat, 1999)

$$Wf(s,x) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t)\psi^*(\frac{t-x}{s})dt . \quad (1)$$

Where, x and s are called translation and scale, respectively. Translation is associated with time-shifting and scale is inversely proportional to frequency.

In vibration signal, the existence of faults usually produces transient signatures that can be treated as singularities. Thus a signal processing technique that ignores the regular part of a signal and focuses on the transient part should have more potential in characterizing these fault-related components. Mallat and Hwang explored this problem (Mallat and Hwang, 1992) and they proposed singularity detection with wavelet method. According to their conclusions, most of the useful information in a signal is contained in the local maxima of the continuous wavelet transform modulus. By examining the asymptotic decay of wavelet modulus maxima from coarser scale to finer scale, the strength of the singularity can be characterized by Lipschitz exponent α, or Holder exponent in some other literatures.

2.1 Characterization of singularity with wavelet

To understand the concept of Lipschitz exponent α, let's firstly look at the definition of it. Lipschitz exponent is a term that can give a quantitative description of function regularity. A function $f(x)$ is Lipschitz exponent α ($n< \alpha \le n+1$) at x_0, if following condition can be satisfied

$$|f(x)-P_n(x-x_0)| \le A|x-x_0|^{\alpha} \quad \text{for} \quad |x-x_0|< h_0 . \tag{2}$$

Where, A and h_0 are two non-negative constants and $f(x)$ is a finite energy function. That is, $f(x) \in L^2(R)$. $P_n(x)$ is a polynomial of order n. Lipschitz regularity of $f(x)$ and x_0 is defined as the superior bound of all values α such that $f(x)$ is Lipschitz α at x_0. Function $f(x)$ that is continuously differentiable at a point is Lipschitz 1 at this point. If the Lipschitz regularity α of $f(x)$ at $x = x_0$, satisfies $n< \alpha <n+1$, then $f(x)$ is n times differentiable at x_0 but its nth derivative is singular and α characterizes this singularity.

In the singularity analysis with wavelet, an important issue is the selection of wavelet with vanishing moment N+1. A wavelet function $\psi(x)$ is said to have N+1 vanishing moments if it is orthogonal to polynomials up to order N, that is

$$\int_{-\infty}^{+\infty} x^k \psi(x)dx = 0, for 0 \le k < N+1 . \tag{3}$$

Then, the wavelet transform of $f(x)$ using $\psi(x)$ at the location x_0 can eliminate those polynomials up to order N. Another important issue in wavelet transform is that a wavelet with N+1 vanishing moments can be written as the N+1th order derivative of the signal $f(x)$ smoothed by a smoothing function $\theta(x)$ in the form

$$Wf(s,x) = f(x)*\psi_s(x) = s^n \frac{d^n}{dx^n}(f*\overline{\theta}_s)(x) \text{, with}$$
$$\overline{\theta}_s(x) = (1/s)\theta(x/s) . \tag{4}$$

Therefore, it is possible to examine any rate of change of the signal amplitude by selecting a suitable wavelet function, because the wavelet transform is a smoothed derivative of the signal at various scales.

Suppose that $\psi(x)$ has N+1 vanishing moments, Mallat and Hwang (1992) proved that if function $f(x)$ is Lipschitz α at x_0, $N< \alpha <N+1$, then there exists a constant A such that for all points x in a neighborhood of x_0 and any scale s,

$$|Wf(s,x)| \le A(s^{\alpha} + |x-x_0|^{\alpha}) . \tag{5}$$

Define modulus maxima as any point (s_0, x_0) such that $|Wf(s_0, x)|$ is a local maxima at $x = x_0$. Singularities can be identified by the presence of modulus maxima. If there exists a scale $s_0 > 0$, and a constant C, such that for $x\in (a, b)$ and $s< s_0$, all the modulus maxima of $Wf(s, x)$ belong to a cone defined by

$$|x-x_0| \le Cs , \tag{6}$$

then at each modulus maxima (s, x) in the cone defined by Eq. (6),

$$|Wf(s,x)| \le As^{\alpha} , \tag{7}$$

which is equivalent to

$$\log_2 |Wf(s,x)| \le \log_2 B + \alpha \log_2 s . \tag{8}$$

Where, $B = A\,(1+ C^{\alpha})$.

2.2 Definition of lipschitz exponent function

Eq. (8) gives an asymptotic relation between wavelet transform and Lipschitz exponent α, and a lot of methods have been proposed to estimate α. The simplest case (Struzik, 2001) uses

$$\alpha = 2^{\frac{1}{m-1}\sum_{s=1}^{m-1}\log_2(\frac{Wf(s+1,x)}{Wf(s,x)})} , \tag{9}$$

where m is the length of maxima line that propagates from coarser scale to finer scale. However, it is not appropriate because this method only utilizes the first and last point of maxima line while discards all the other points. Peng et al proposed another method (Peng et al., 2002), in which the object function is:

$$\sum_s [\log_2 |Wf(s,x)| - \log_2 A - \alpha * \log_2 s]^2 . \tag{10}$$

Through minimizing Eq. (10) at all scales, we can obtain A and α. Then, the problem of α estimation is transformed into optimization and this can be solved by non-linear least-square method. Moreover, Hong et al. (2002) used linear regression technique which is an easy way to implement. Although linear regression method is a simplification of Eq. (8) that gives a conservative result, it is applicable since we only need a description of change of Lipschitz exponent. In fact, such a method has been applied by other researchers (Hong et al., 2002; Robertson et al., 2003) and achieves good results. With this method, we have

$$\alpha(x)=\frac{\sum_{s=1}^{m}\left(\log_2|Wf(s,x)|*\log_2 s\right)-\frac{\left(\sum_{s=1}^{m}\log_2|Wf(s,x)|\right)\left(\sum_{s=1}^{m}\log_2 s\right)}{m}}{\sum_{s=1}^{m}\left(\log_2 s\right)^2-\frac{\left(\sum_{s=1}^{m}\left(\log_2 s\right)\right)^2}{m}}. \tag{11}$$

With Eq.(11), we propose a new concept, Lipschitz exponent function $Lp(x)$. $Lp(x)$ is a function that describes the change of Lipschitz exponent value Lp (or α) along the temporal axis x. Usually, the temporal variable x is consistent with sampling point in one revolution (for rotating machine). Given a wavelet transform of function $f(x)$, $Wf(s, x)$ is a two-dimensional time-scale representation in which x represents a different time point in the signal while the other one denotes a different frequency scale s. Below is the procedure of extracting Lipschitz exponent function $Lp(x)$,

1. Let x=1 and take the first column as $V(s)=\log_2|Wf(s,x)|$ for s=1,…, l, which represents the frequency spectrum of the signal at the corresponding time point x=1;
2. Calculate Lipschitz exponent α using $V(s)$ and Eq.(11);
3. Let $Lp(x)=\alpha(x)$, $x=x+1$ and $V(s)=\log_2|Wf(s,x)|$;
4. If x reaches its maximum value, then end the program; else go to Step 2;
5. Finally we get Lipschitz exponent function $Lp(x)$.

In case there exists a fault in machinery, it produces singularity in vibration signal whose Lipshcitz exponent varies significantly and can be explicitly observed by plotting the corresponding $Lp(x)$. Figure 2 is an example to demonstrate this phenomenon. They are two pieces of signals collected from the case of gearbox. Comparison between 2(a) and 2(b) shows the obvious increase of Lp around sampling point x=700. Analysis after the experiment tells us the reason for this change is due to gear tooth broken, which demonstrates the potential ability of singularity detection (or fault detection in this example) with Lipschitz exponent function $Lp(x)$.

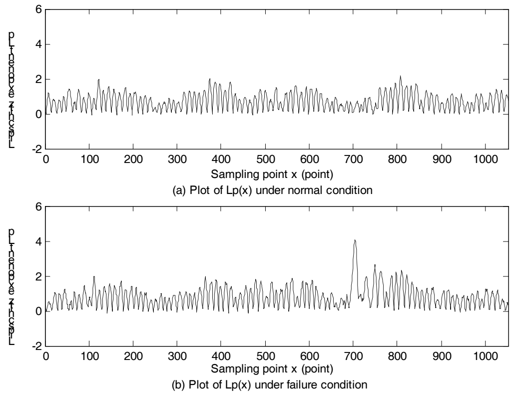

Fig. 2. Plot of $Lp(x)$ under two health conditions in one revolution.

3. Condition description: Definition of health index

The proposed Lipschitz exponent function $Lp(x)$ in the previous section provide us a novel way of feature extraction for machinery fault detection. However, to associate such a technique with CBM decision-making imposes a further constraint on its development since optimal maintenance decision-making is in need of quantitative description of the health status of the target to be maintained. Kurtosis has been frequently used as a quantitative indicator for evaluating gear fault severity. For example, McFadden (1986) utilized kurtosis to evaluate the severity of a fault, which is calculated based on the phase modulation trace over an entire revolution of the target gear.

It should be noted that kurtosis may not be proportional to the advancement of gear fault, in particular when multiple teeth are involved in a fault, a dramatic decrease of kurtosis is usually present. However, such a drawback of kurtosis is not critical to practical condition monitoring and maintenance since responsible maintenance personnel should schedule shutdown immediately at the early stage of a fault. Based on the above analysis, we proposed a new Kurtosis based Health Index (KHI) by calculating the kurtosis of $Lp(x)$. That is,

$$KHI(t) = kurtosis(Lp_t(x)) . \quad (12)$$

Where, subscript t means the machine running time and $Lp_t(x)$ is the Lipschitz exponent function $Lp(x)$ at time t. Since we will use full lifetime gearbox vibration data to investigate the proposed KHI, and vibration data are collected and numbered at a certain frequency (e.g., once an hour) along with the running of target, time t is consistent with file number and is represented by file number (1, 2,… n,…).

In order to investigate the performance of this new index KHI, a comparison with indexes proposed by Miller, Miao and Viliam (Miller, 1999; Miao and Viliam, 2005) will be conducted in this paper using the same gearbox vibration data provided by Pennsylvania State University. Miller (1999) proposed a gear state indicator termed fault growth parameter (FGP) using comblet for the decomposition of vibration signal. This FGP is defined as the percentage of the residual error signal which exceeds three standard deviations calculated from the baseline residual error signal taken when the run began, that is,

$$FGP1(t) = \frac{100}{L}\sum_{i=1}^{L}\left[I(r_i(t) > \overline{r} + 3\sigma_0)\right. \\ \left. + I(r_i(t) < \overline{r} - 3\sigma_0)\right] , \quad (13)$$

where L is the length of residual error signal, $r_i(t)$ is the residual error signal at time t, $\overline{r}$ is the mean value of the residual signal at time t, σ_0 is the "baseline" standard deviation and $I(*)$ is the indicator function.

Furthermore, Miao and Viliam (2005) proposed two health indexes and the definitions of them are

$$FGP2(t) = \sum_{x} | m_t(x) - m_0(x) | , \quad (14)$$

$$FGP3(t) = \lambda * FGP2(t) + (1-\lambda) * FGP3(t-1) . \quad (15)$$

Where, $m_0(x)$ is the reference modulus maxima distribution function, $m_t(x)$ is the modulus maxima distribution at time t and $0 < \lambda \le 1$ is a constant. FGP3 is an EWMA (exponentially weighted moving average) statistic of FGP2. For more details, readers can refer to Miao and Viliam (2005).

4. Experimental validation

The proposed health index, KHI, is applied to two sets of vibration signals collected from a mechanical diagnostics test bed (see Fig. 3) with the same type of single reduction helical gearbox as its health condition progresses from brand new to driven gear failure (tooth broken). These vibration data are provided by Applied Research Laboratory (ARL) at Pennsylvania State University. For each test run analyzed in this paper, there are a number of unequally-spaced inspections performed in the process of test rig running. Each inspection provides a piece of signal collected in a 10-second window at sampling rate of 20kHz. The signal is saved on a computer as a data file and numbered consequently. Given the mechanical specification of the gearbox in Table 1, the length of signal in one revolution of driven gear can be computed and in these two test runs (TR#5 and TR#10), it has 1051 sampling points per revolution. To compare the performance of KHI with other health indexes (FGP1, FGP2 and FGP3), we select 20 data files right before the occurrence of gear failure and 10 data files right after failure in each test run. Thus, there are totally 30 data files in each study. In this paper, the wavelet function for signal processing and singularity analysis is the derivative of Gaussian function, whose vanishing moment is 2.

4.1 Comparison in TR#5

TR#5 was shut down due to two adjacent teeth bro-

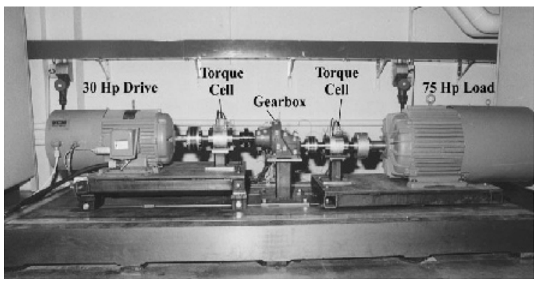

Fig. 3. Echanical diagnostic test bed.

Table 1. Gearbox information of TR#5 and TR#10.

Gearbox ID#	DS3S0150XX
Make	Dodge APG
Model	R86001
Gear Ratio	1.533
Contact Ratio	2.388
Number of Teeth (Driven gear)	46
Numver of Teeth (Pinion gear)	30
Meshing frequency	875.53Hz

ken of the driven gear at the end of the test. The total number of running hours is 127.4 hours, including 83 data files. Based on the analysis by ARL (Miller, 1999), the beginning of incipient fault starts at data file 72. Thus, we choose data files with number from 52 to 71 as normal condition and number from 72 to 81 as failure condition. That is, the vibration data used in this study have file number from 52 to 81.

Figure 4 shows the comparison result of four health indexes using the data in TR#5. In this section, we use horizontal dashed line to approximately represent threshold level of machine health condition. That is, the health indexes less than threshold level are below the dashed line and those greater than the level are above the line. In addition, a vertical dotted line is plotted in each graph to represent the beginning of machine failure. Based on the results of 4(a), 4(b), 4(c) and 4(d), three of them (KHI, FGP1 and FGP2) can accurately identify the incipient gear tooth failure while FGP3's ability of failure identification is poorer that others since the value of FGP3(71) is very close to FGP3(72). However, the performance of KHI is better than others because the fluctuation of FGP1 and FGP2 is so large that sometimes it may result in an incorrect conclusion. Furthermore, the selection of threshold level is another reason that proves that KHI is better than FGP1 and FGP2. The determination of threshold level for FGP1 and FGP2 is difficult and we can only draw a conclusion based on after-the-event analysis, while for KHI, it is widely accepted that kurtosis greater than 3 can be used as criterion and threshold level (McFadden, 1986; Zhan et al., 2006). Thus, we can conclude that KHI performs better than three other indexes.

4.2 Comparison in TR#10

TR#10 was shut down due to a distributed gear teeth broken of the driven gear at the end of the test. The total number of running hours is 189.25 hours, including 148 data files. Based on the analysis by ARL (Miller, 1999), the beginning of incipient fault starts at data file 38. Thus, we choose data files with number from 18 to 37 as normal condition and number from 38 to 47 as failure condition. That is, the vibration data used in this study have file number from 18 to 47.

Figure 5 shows the comparison result of four health indexes using the data in TR#10. As shown in Fig. 5(a)~(d), the occurrence of incipient failure is identified at file number 38, which is consistent with the conclusion provided by ARL. We use dashed line and dotted line as threshold level and boundary. In this comparison, the performance of FGP3 is poorest

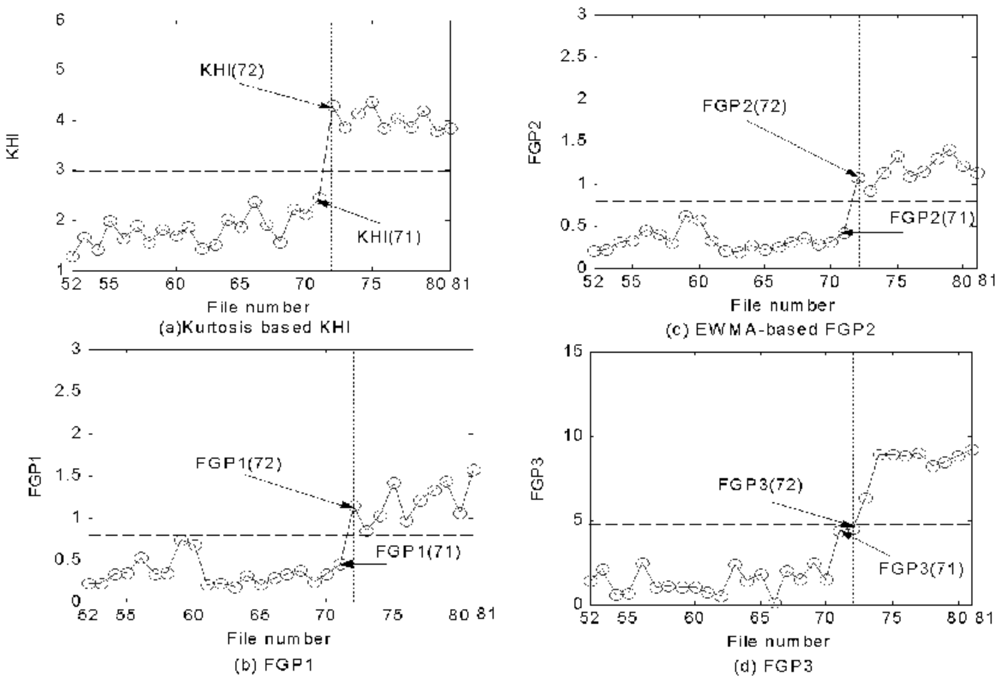

Fig. 4.Comparison study of four health indexes using TR#5.

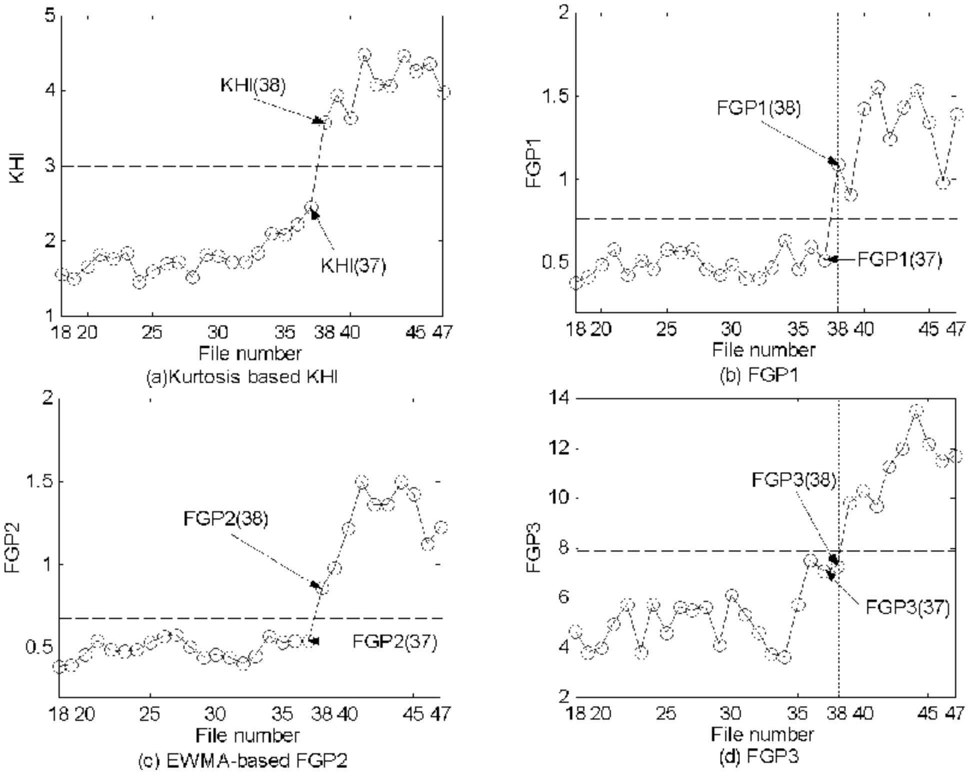

Fig. 5. Comparison study of four health indexes using TR#10.

since it cannot accurately identify the occurrence of failure. Similarly, we get the same conclusion that KHI is the best health index in that it is more stable than FGP1 and FGP2 under same health condition.

5. Conclusions and future work

In this paper, we applied a new signal processing technique, singularity analysis with wavelet, to gearbox fault detection problem. A novel concept, Lipschitz exponent function $Lp(x)$, is proposed in this research and a health index KHI is defined based on Lipschitz exponent function. A comparative study of proposed health index with previous work is conducted and analysis results show that KHI has excellent performance in gearbox fault detection.

Research on extraction of health index provides a quantitative description of machine health condition. This information can also be used for machine remaining life prediction, which is an important area that has been investigated by many researchers. During next step, we will go on our work along this direction.

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